# The Language of Special Relativity[*]

Thomas Müller

## 1. Introduction

The Special Theory of Relativity is often interpreted as forcing us to accept a conceptual revolution in our thinking about space, time, and the temporal distinction between past, present, and future. Most importantly, it is claimed that the notion of absolute simultaneity that is embodied in our pre-relativistic thinking about time must be abandoned. Thus, our natural language use of linguistic tense appears to be ill-founded. This constitutes a problem for philosophical analysis: Given that our natural language talk is based on a (supposedly erroneous) conception of absolute simultaneity, can we analyze and clarify such talk? *Prima facie*, our natural language use of tenses is well captured by tense logic. Can we adjust the machinery of that logical framework such as to meet the relativistic challenge?

In my paper, I will first sketch the project of tense logic as a so-called A-theory of time (Section 2). I will then state the relativistic challenge (Section 3). Finally, I will try to answer the challenge by extending the formal framework of tense logic, taking a lead from Arthur Prior's idea of a "logic of points of view" (Section 4). Very much in the spirit of tense logic, I will claim that the special theory of relativity is best interpreted as a multi-modal logic.

## 2. Tense logic as an A-theory of time

It is customary to divide philosophical theories of time into two camps: so-called A-theories and B-theories. The terminology is taken from an article by McTaggart (1908).[1] A-theories take the distinction between past, present, and future to be metaphysically fundamental, whereas B-theories assume that the relative notions of being earlier than, simultaneous with, and later than are fundamental. The main difference is thus about the question of the status of the present: A-theories assume that the present is real, whereas B-theories deny the reality of the present. *Prima facie*, our natural language talk favors an A-theory, since the distinction between past, present, and future, as embodied in the phenomenon of tense, is pervasive. The dispute between A- and B-theories can thus also be seen as one about the status of linguistic tense; hence the labels "tensers" and

[*] I wish to thank audiences at Bonn and at Bielefeld for helpful discussions.

[1] McTaggart introduces his terminology in the context of an argument that supposedly shows that time, which is necessarily an A-theoretic concept, is unreal due to an inherent contradiction in the A-theoretic view. For a decisive criticism of McTaggart's argument cf. Prior (1967), Chap. I. Cf. also Müller (2002), Chap. 4.2.

"de-tensers" for A- and B-theorists, respectively.[2] A-theorists are also sometimes called "presentists" because they favor a metaphysically fundamental role of the present.

Tense logic was originally conceived by Arthur Prior in the 1950s as an attempt at capturing our natural language use of tenses in a formally rigorous way, using the resources of modal logic (cf. Prior 1957). In tense logic, one considers two temporal operators: "P", "it was the case that", and "F", "it will be the case that". These operators are treated syntactically and semantically like standard weak modal operators; the corresponding strong operators are often symbolized as "H" ("it has always been the case that") and "G" ("it is always going to be the case that"). In tense logic, atomic sentences are considered to be in the present tense. The analogy with modal logic is complete: In modal logic, plain $p$ is evaluated at the actual world, and in tense logic, plain $p$ is evaluated at the actual time, i.e., at the present. By prefixing sentences with the temporal operators, other tenses are formulated, again in full analogy with the formation of modalized sentences in modal logic.

Tense logic is an inherently A-theoretic project, since the atomic sentences refer to the present. This is in sharp contrast to the treatment of time in other formal settings. E.g., Quine (1960), who was opposed to all kinds of modal logic, showed how the resources of predicate logic naturally lead to a B-theoretic conception of time. On the other hand, Prior, stressing the A-theoretic commitment of tense logic, even claimed that the tenses should have an ontological interpretation: "[T]he present simply *is* the real considered in relation to two particular species of unreality, namely the past and the future" (Prior 1970, 245).

It is often argued that the tense-logical project, and indeed any A-theoretic conception of time, has been proved invalid by the empirical success of the special theory of relativity. We will now consider that relativistic challenge.

## 3. The relativistic challenge

The special theory of relativity (STR) was formulated by Einstein in 1905, based on previous work by Lorentz and Poincaré. As the title of Einstein's famous paper, "On the electrodynamics of moving bodies" (1905), suggests, the theory was initially devised to solve a problem in electrodynamics: While everybody believed (and empirical findings were later to demonstrate)[3] that electromagnetic phenomena are independent of the inertial frame in which they are observed, in Maxwell's theory, certain phenomena are described radically differently in different inertial frames. Einstein offered a solution to this problem by basing his theory of relativity on two postulates: (1) The laws of physics have the same form in all inertial frames, (2) the speed of light is the same in all inertial frames. Given these postulates, it turns out that in considering a change of inertial

[2] To the best of my knowledge, that terminology was first used by Massey (1969, 18f.).

[3] In this context it should be observed that the story of the Michelson-Morley experiment as an *experimentum crucis* in favor of relativity theory is a myth. Cf. Collins and Pinch (1998), Chap. 2 and Appendix.

frame, one has to use local coordinates that differ not just in the spatial, but also in the temporal component. Thus, e.g., a temporal difference of $\Delta t = 0$ (simultaneity) in one inertial frame may correspond to either a positive or a negative $\Delta t'$ in another inertial frame. This obviously poses a problem for the notion of simultaneity. Einstein originally claimed that given relativity, simultaneity has to be relativized to an inertial frame: The seemingly natural notion of absolute simultaneity literally makes no sense (cf. Einstein 1905 and 1917). Thus, our natural language use of tenses appears to be ill-founded. This semantical claim needs to be backed by a strong form of verificationism that Einstein initially subscribed to, but was later to abandon at least in part. However, Einstein's teachings have been influential: It is now common lore that it is senseless to speak about absolute simultaneity.

Given this empirical background, the tense logical project has been accused of being "ill-advised because grounded in bad physics" (Massey 1969, 31). An argument to this effect has been used by various authors. Consider the following version taken from Hugh Mellor's recent book, *Real Time II*:[4]

> Take the presentist view, that only the present exists. On this view the only events on Sirius which now exist are those that are simultaneous with your present reading of this very sentence. Yet in relativity, as we have seen, if Sirius is ten light years away, which events these are will vary from frame to frame over a twenty-year span. And if nothing in the world makes one frame right, nothing will ever make any shortlived Sirian event present as opposed to past or future. It will be present in some frames, past in some and future in some, and that is all there is to it. But that can hardly be all there is to existence.
>
> This is not only a problem for presentists. Those who think that the past and present exist, but the future does not, face it too. So do those who, while they reject *A*-facts, think that not even *B*-facts about events exist until they do. On these views also the existence of events depends on simultaneity, either with what is present here or with each other. So for these views too the existence of remote events can be a matter of fact at any earthly time *t* only if their simultaneity with events here at *t* is also a matter of fact, which in relativity it is not.
>
> In short, adding relativity to any of these views makes existence relative to places as well as times, which is incredible. (Mellor 1998, 56f.)

Mellor presents an argument with four premises (a)–(d),[5] concluding that tense-logic is ruled out:

a. Tense logic proposes an ontological distinction between past, present, and future. "Being present relative to" must be an equivalence relation and hence, transitive.
b. Given STR, the only definable transitive relations are trivial (identity and the universal relation).

[4] A more detailed exposition of the argument may be found in Müller (2002), Chap. 4.4.

[5] Premises (a) and (b) are taken from unproblematic background assumptions that Mellor shares. Premise (c), which is required in order for the argument to be logically valid, is not explicitly stated in the quoted passage either, but it is clear from context that Mellor assumes (c).

c. The ontological distinction that tense logic presupposes must be definable on the basis of STR.
d. The relativized notion of presentness offered by STR cannot be the basis for tense logical presentness.
e. Thus, tense logic and other A-theories are ruled out by STR.

This argument is logically valid. As to its premises, (a) is true, as witnessed by the quote from Prior at the end of Section 2. (b) is also true (cf., e.g., Clifton and Hogarth 1995).[6] However, premises (c) and (d) leave loopholes for the tense logician.

## 4. The proposed solution

In what follows, I will describe a broadly tense-logical way of denying the relativistic challenge by attacking its premise (d). Before that, however, it will be good to point out some ways of criticizing premise (c).[7]

Premise (c) can be attacked on methodological grounds. The question that the relativistic challenge is concerned with is certainly a metaphysical one, i.e., the dispute between A- and B-theories of time. In order for premise (c) to function in this metaphysical setting, it is necessary to grant the special theory of relativity a metaphysical status, i.e., the status of a metaphysical theory of space and time. However, STR is an empirical theory. It is true that STR is extremely well confirmed *within its limits of application*, ranging from a prediction of decay times for cosmic radiation to the equivalence of mass and energy exploited in nuclear fission. However, it is equally clear that *the world is not a model of STR*. There are real phenomena such as gravitation or quantum mechanical experiments that fall outside the scope of STR. This is not meant to be a criticism of STR itself, it is only meant to point out that it is illigitimate to use that theory in assessing metaphysical claims. If STR is given a grand metaphysical reading, the theory is just *false*. — Apart from this methodological worry, a technical result by Rakić (1997) shows that it is possible to add an equivalence relation (signifying absolute simultaneity) as a conservative extension to the structure of Minkowski spacetime. This technical result meshes well with the fact that while STR does not offer a notion of absolute simultaneity, it does not rule out such a notion either (unless one assumes verificationism or the like). Thus, the formal resources for a tense logic based on an absolute notion of simultaneity are available, and there are even some physical (e.g., cosmological or quantum mechanical) motivations for an absolute notion of simultaneity available.[8] However, in what follows, I will try to sketch the systematically more pleasing option of attacking premise (d) of the above argument.

6 Similar results were obtained by Stein (1991, 149f.) and van Benthem (1991, 25).
7 For a fuller exposition of possibilities for denying premise (c), cf. again Müller (2002), Chap. 4.4.
8 Cf. the previous note.

The proposed solution is a 'logic of points of view' (LPV; cf. Prior 1968, 133ff.), based on Prior's idea of so-called egocentric logic.[9] In his article, "Tense logic and the logic of earlier and later" (Prior 1968, 116–134), Prior first sketches a response to the relativistic challenge that denies premise (c) of the argument, thus favoring an absolute conception of simultaneity (cf. Prior 1968, 133f.). After that, he gives the following outline of a tense logic that denies premise (d):

> We can, all the same, develop the logic of 'points of view' on the basis of a syntax which does *not* thus suggest that there is a 'real' (though only partly knowable) course of events which represents these various systematically related appearances. We might describe this alternative syntax in a very general way as follows: instead of using the plain *p* for a quite impersonal 'It is (really) the case that *p*', we use it for 'It appears (or is the case) from *this* point of view that *p*', or 'It is the case with this person or particle that *p*'. That is, the prefix 'It appears from *this* point of view that–' or 'It is the case with *this* person or particle that–' is one which has the same sort of vacuity in this language as 'It is *now* the case that–' has in ordinary tense logic; it does not need to be expressed, but is understood in all that we say. We then describe what appears to be the case from other points of view, or what is the case with other persons or particles, by using quasi-modal operators which take us from 'this' point of view or particle to the other ones, very much like operators like 'It will be the case that–' take us to other 'nows' from 'this' now. (Prior 1968, 134)

Even though Prior continues the quoted passage by stating that he is not convinced that this approach will lead to satisfactory results, it turns out that his sketch can be given a fruitful formal reading.[10]

The non-modalized formulas $\phi$ of LPV are to be interpreted as "In this frame of reference, $\phi$". (This is similar to tense logic, where plain $\phi$ is interpreted as "It is now the case that $\phi$".) $\phi$ can be any formula of predicate logic. We next introduce a group $L$ of modal operators that describe changes of the point of view ("It appears from another point of view that–"). In relativity theory, points of view are naturally taken to be inertial frames, and changes between these frames are described by Lorentz transformations. Since we need to consider not only rotations and boosts, but also translations, we have to use the full inhomogeneous Lorentz group, which is also known as the Poincaré group. Thus, for every element (Lorentz transformation) $l$ of the 10-parameter (proper orthochronic)[11] Poincaré group $P$, we introduce a modal operator $[l]$.[12] These modal

9 For Prior's notion of "egocentric logic" cf. his (1968, 135–144) and Prior and Fine (1977, 28–50).

10 An extended exposition of a previous formalization of *both* of Prior's approaches to a "logic of points of view" may be found in Müller (2002), Chap. 4.5. The system shown here is an extension of what I called there "ISL" (for "idealistische Standpunkt-Logik").

11 We will ignore spatial and temporal inversions since these do not correspond to changes of the point of view—there is no point of view from which, e.g., a right-hand glove fits a left hand.

12 By taking the points of view to be elements of the Poincaré group, we subscribe to a uniform choice of synchronization (e.g., $\varepsilon = ½$). One may wish to individuate "points of view" more finely by allowing for different individual synchronization conventions; cf. Sarkar and Stachel (1999) vs. Malament (1977). As long as a group structure of the "points of view" is preserved, this will not lead to any changes in the semantics.—Thanks to Holger Lyre for a discussion of this point.

operators turn out to be self-dual (i.e., they are both weak and strong operators) since the group multiplication, which will form the "accessibility relation", is functional. The set of well-formed formulas (wff) of LPV is thus defined as follows (we leave out equality and functions for brevity's sake; the standard definitions may be used to add these):

W0 A term is either a constant symbol or a variable.
W1 If $A$ is an n-ary predicate symbol and $t_1,\dots,t_n$ are terms, then $A(t_1,\dots,t_n)$ is a wff.
W2 If $\phi$ is a wff, then $\neg\phi$ is a wff.
W3 If $\phi$ and $\psi$ are wff, then $(\phi \wedge \psi)$ is a wff; similarly for $\vee$, $\rightarrow$, $\leftrightarrow$.
W4 If $\phi$ is a wff and x is a variable, then $\forall x\phi$ and $\exists x\phi$ are wff.
W5 If $\phi$ is a wff and $l$ is an element of $P$, then $[l]\phi$ is a wff.
W6 Nothing else is a wff.

The semantics for the predicate logical part takes a universe of discourse $U$ to be a set of connected subsets of worldlines in Minkowski spacetime. For simplicity's sake, we consider bounded subsets only, so that each $u \in U$ may be taken to be parameterized by the interval [0,1]: each $u$ is a function from [0,1] to Minkowski spacetime.[13] A model $M$ consists of a universe $U$, an interpretation $I(c)$ for each constant symbol $c$ (so that $I(c) \in U$), and an interpretation $I(A)$ for each predicate symbol $A$. With respect to the latter, the semantics need to differ from the usual ways of predicate logic: While in predicate logic, each n-ary predicate symbol is interpreted as a set of n-tuples of members of the universe, we wish to consider predicates that do not necessarily apply to individuals throughout their whole history, but which may apply at some times and not at others, which is the essence of change. E.g., a rabbit (its world line taken to be an element $u$ of $U$) may have a dull or an interesting life (i.e., the predicates "dull" or "interesting" may apply to $u$ as a whole), but the rabbit may be asleep at some times and awake at other times (the predicates "asleep" and "awake" do not apply to $u$ as a whole, but do apply to $u$ at some times). If we do not wish to deny change, we need to use a non-standard semantics for predicates. Thus, a unary predicate $A$ is interpreted not by a subset of $U$, but by a subset of $U \times [0,1]$; $\langle u,\tau\rangle \in I(A)$ means that the predicate $A$ applies to individual $u$ at the spacetime point $u(\tau)$. For n-ary predicates, the interpretation is defined analogously. A formula is evaluated in a model $M$, supplemented with a valuation $v$ (specifying a member of $U$ for each variable), in a frame of reference $f \in P$. A frame of reference uniquely determines a present time hypersurface, viz., the set of all points in Minkowski spacetime for which $t = 0$ in local coordinates. (These points are simultaneous with the origin of the frame of reference.) Atomic formulas are interpreted in the following way (exemplified for a one-place predicate $A$ and a variable $x$, so that $v(x) \in U$ is the individual for which $x$ stands):

[13] This assumption will save us the trouble of repeating many clauses with (0,1], [0,1) and (0,1) in place of [0,1]. It should be obvious how the assumption can be avoided. It is certainly not meant to rule out unending individuals metaphysically.

$M,v,f \models A(x)$ iff[14] there is $\tau \in [0,1]$ such that the spacetime point $(v(x))(\tau)$ belongs to the present time hypersurface of $f$ and $\langle v(x),\tau\rangle \in \mathrm{I}(A)$.

For the junctors and quantifiers, the standard definitions are used, e.g.,

$M,v,f \models (\phi\wedge\psi)$ iff $M,v,f \models \phi$ and $M,v,f \models \psi$.

The semantic clause for the modal operators is as follows:

$M,v,f \models [l]\phi$ iff $M,v,l(f) \models \phi$,

where $l(f)$ is the frame of reference $f$ transformed by the Lorentz transformation $l$. Due to the group structure of $P$, the modal operators have the following desirable features: (1) By the properties of group multiplication, two adjacent modal operators combine to a single operator: provably, $[l][l']\phi$ iff $[l'*l]\phi$. (2) Since every element of $P$ has an inverse, it is always possible to "undo" a change of point of view: provably,

$[l][l^{-1}]\phi$ iff $[l^{-1}][l]\phi$ iff $\phi$.[15]

The logical system LPV that has been presented so far can easily be extended such as to gain a greater expressiveness. Tense operators, e.g., Prior's future tense operator "F", may be introduced by reading $\mathrm{F}\phi$ as a generalization:

$M,v,f \models \mathrm{F}\phi$ iff there is a purely temporal translation $l\in P$ with $\Delta t > 0$ s.t. $M,v,l(f) \models \phi$;

past tense "P" is treated symmetrically (with $\Delta t<0$). The frame-invariant structure of Minkowski spacetime can similarly be captured by generalizing over the whole homogeneous Lorentz group, yielding the standard relativistic "causal tense operators" studied, e.g., by Goldblatt (1980). As a further extension, a notion of dates may be implemented by specifying for each frame a "standard" element of $P$ that transforms that frame to, e.g., "1st January 2001, 0:00, at rest at Greenwich".[16]

---

[14] "Iff" is short for "if and only if".

[15] This feature is especially important when it comes to explaining communication in an indexical, relativistic setting. Cf. Müller (2004).

[16] Taking this transformation to be frame-relative allows for an internalist perspective on dates that captures phenomena like slow clocks or oversleeping à la Rip van Winkle. Cf. Müller (2002), Chap. 4.3.3 for arguments in favor of the approach. For more details on extending LPV, cf. Chap. 4.5 of that work.

## 5. Conclusion

In which sense does LPV provide a solution to the relativistic challenge? According to premise (d) of the argument (cf. section 3 above), a relativized notion of presentness is considered inappropriate, since it is allegedly unable to provide a basis for a relativistic tense logic. The system LPV shows that there is no *formal* reason why one should hold on to premise (d). Furthermore, in LPV, there is no need for an explicit relativization of the present—the indexical structure of our natural language use of the present tense is fully and adequately captured by the system.

Some B-theorists may still not be convinced. What about the ontological status of the present? What kind of "ontological distinction" can the present have according to LPV? Is the notion of the present employed in LPV "objective"? I have two replies to offer. First, objectivity, understood as intersubjective agreement between different frames of reference, is fully guaranteed: By knowing "where my discourse partner is", i.e., knowing which transformation $l$ will lead to her point of view, her $\phi$ can be understood as my $[l]\phi$. There can be universal agreement as to matters past, present, and future. Conceding that a certain feature is indexical does not mean that it is "relative". Secondly, if one wishes to hold on to a "thicker" metaphysical notion of the reality of present (as Prior would have it in most of his writings), then the way is open to introduce an absolute rest frame to LPV. Whether one wishes to hold on to a preferred frame or not, LPV can capture our natural language use of tense in a relativistically correct way. Thus, LPV provides a basic language for the special theory of relativity.

## Bibliography


Clifton, R. and Hogarth, M. (1995): "The definability of objective becoming in Minkowski spacetime", *Synthese* 103: 355–387.

Collins, H. and Pinch, T. (1998): *The Golem. What You Should Know About Science*. (2nd edition.) Cambridge: Cambridge University Press.

Einstein, A. (1905): "Zur Elektrodynamik bewegter Körper", *Annalen der Physik* 17: 891–921.

Einstein, A. (1917): *Über die spezielle und die allgemeine Relativitätstheorie*. Braunschweig: Vieweg.

Goldblatt, R. (1980): "Diodorean modality in Minkowski spacetime", *Studia Logica* 23: 219–236.

Malament, D. (1977): "Causal theories of time and the conventionality of simultaneity", *Noûs* 11: 293–300.

Massey, G. (1969): "Tense logic! Why bother?", *Noûs* 3: 17–32.

McTaggart, J.M.E. (1908): "The unreality of time", *Mind* 18: 457–484.

Mellor, D.H. (1998): *Real Time II*. London: Routledge.

Müller, T. (2002): *Arthur Priors Zeitlogik. Eine problemorientierte Darstellung*. Paderborn: Mentis.

Müller, T. (2004): “Communication in a relativistic setting”, in preparation.
Prior, A. (1957): *Time and Modality*. Oxford: Oxford University Press.
Prior, A. (1967): *Past, Present and Future*. Oxford: Oxford University Press.
Prior, A. (1968): *Papers on Time and Tense*. Oxford: Oxford University Press.
Prior, A. (1970): “The notion of the present”, *Studium Generale* 23: 245–248.
Prior, A. and Fine, K. (1977): *Worlds, Times and Selves*. London: Duckworth.
Quine. W.V.O. (1960): *Word and Object*. Cambridge, MA: MIT Press.
Rakić, N. (1997): “Past, present, future, and special relativity”, *British Journal for the Philosophy of Science* 48: 257–280.
Sarkar, S. and Stachel, J. (1999): “Did Malament prove the non-conventionality of simultaneity in the special theory of relativity?”, *Philosophy of Science* 66: 208–220.
Stein, H. (1991): “On relativity theory and the openness of the future”, *Philosophy of Science* 58: 5–23.
van Benthem, J. (1991): *The Logic of Time*. (2nd edition.) Dordrecht: Kluwer.